Centro de Estudios Científicos

CECS-PHY-04/07

Finite action principle for Chern-Simons AdS gravity

P. Mora[1], R. Olea[2], R. Troncoso[3] and J. Zanelli[3]

[1] *Instituto de Física, Facultad de Ciencias Iguá 4225, Montevideo, Uruguay.*
[2] *Departamento de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile.*
[3] *Centro de Estudios Científicos (CECS), Casilla 1469, Valdivia, Chile.*

Abstract: A finite action principle for Chern-Simons AdS gravity is presented. The construction is carried out in detail first in five dimensions, where the bulk action is given by a particular combination of the Einstein-Hilbert action with negative cosmological constant and a Gauss-Bonnet term; and is then generalized for arbitrary odd dimensions. The boundary term needed to render the action finite is singled out demanding the action to attain an extremum for an appropriate set of boundary conditions. The boundary term is a local function of the fields at the boundary and is sufficient to render the action finite for asymptotically AdS solutions, without requiring background fields. It is shown that the Euclidean continuation of the action correctly describes black hole thermodynamics in the canonical ensemble. Additionally, background independent conserved charges associated with the asymptotic symmetries can be written as surface integrals by direct application of Noether's theorem.

1. Introduction

The relevance of having a finite action for gravity has been recognized since the early days of black hole thermodynamics [1]. In the presence of a negative cosmological constant, this problem has gained new interest in view of the AdS/CFT correspondence (see [2] and references therein). A first approach to regularize the action involved choosing a reference background with suitable matching conditions [3]. However, as these conditions depend on the topology and the asymptotic behavior of the solution, the background must be selected case by case. Moreover, such a choice is not necessarily unique and in some cases, it might prove impossible to find one.

An interesting alternative to tackle this problem, inspired by the AdS/CFT correspondence, is the background independent counterterms method [4] (see also [5]), which involves the addition of a linear combination of invariants of the intrinsic geometry at the boundary. However, as the spacetime dimension increases there is a plethora of possible terms (see e.g., [6, 7]), and the full series for any dimension is unknown.

On the other hand, in dimensions greater than four, gravity can be described by a generalization of the Einstein-Hilbert action, whose Lagrangian is an arbitrary linear combination of the dimensional continuation of the Euler densities [8, 9]. These so-called Lovelock Lagrangians still lead to second order field equations for the metric. In odd dimensions there exist special combinations of these terms that allows to write the Lagrangians as Chern-Simons densities [10] which possess a number of interesting features. In particular, apart from general coordinate invariance, these theories possess an enhanced local symmetry and their local supersymmetric extensions have been constructed in three [11, 12], five [13], and higher odd dimensions [14, 15, 16, 17, 18, 19].

Here we discuss an action principle for Chern-Simons AdS gravity which is background independent for all odd dimensions. As both the background and the counterterm approaches described above become cumbersome when one deals with a Lagrangian containing higher powers in the curvature in higher odd dimensions, we follow a different strategy.

A set of boundary conditions for the geometry is proposed, which makes the Chern-Simons action functionally differentiable at the extremum. The boundary conditions single out a boundary term which is a local function of the fields at the boundary. The resulting action principle requires no background fields, is finite for configurations that satisfy the boundary conditions, and the Euclidean continuation of the action correctly describes the thermodynamics of the system in the canonical ensemble. Furthermore, conserved charges associated with the asymptotic symmetries can be written as surface integrals at infinity by direct application of Noether's theorem without the need for ad hoc regularizations.

In the next section the five dimensional case is analyzed in detail. Section 3 is devoted to generalize the results for any odd dimension. The applications to black hole thermodynamics, and the construction of the conserved charges is carried out in Sections 4 and 5, respectively.

2. Chern-Simons-AdS Gravity in Five Dimensions

Let us consider five-dimensional gravity with a negative cosmological constant and a Gauss-Bonnet term

$$I_5 = -\frac{4\kappa}{l^2}\int\limits_M d^5x\sqrt{-g}\left[R+\frac{6}{l^2}+\frac{l^2}{4}\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2\right)\right]+\int\limits_{\partial M} B_4\ , \tag{2.1}$$

where the relative coefficients in (2.1) have been fixed so that there is a unique negative constant curvature solution (AdS) with radius l. The boundary term B_4 can be found as follows. We require the action to have an extremum under variations of the geometry with a fixed extrinsic curvature at the boundary, i.e.,

$$\delta K_{ij} = 0\ , \tag{2.2}$$

where K_{ij} is fixed in terms of the boundary metric h_{ij} as

$$K_{ij} = \Omega h_{ij}\ , \tag{2.3}$$

for some arbitrary function Ω at ∂M. The remarkable feature of these boundary conditions is that the boundary term can be integrated to be

$$\begin{aligned} B_4 = 2\kappa\sqrt{-h}\Big[&-K\left(\frac{1}{l^2}+\widetilde{R}+\frac{1}{2}\left(3K_{ij}K^{ij}-K^2\right)\right)\\ &+4\widetilde{R}_{ij}K^{ij}+K^i_jK^j_kK^k_i\Big]\ , \end{aligned} \tag{2.4}$$

so that the resulting action principle is background independent. Furthermore, this procedure also ensures the convergence of the action for asymptotically AdS configurations. In Eq. (2.4), $\widetilde{R}_{ij}$, and $\widetilde{R}$ are the Ricci tensor and the curvature scalar of ∂M, respectively. The extrinsic curvature is the Lie derivative along the normal to the boundary, n,

$$K_{ij} = \mathcal{L}_n h_{ij}\ . \tag{2.5}$$

Hence, the condition (2.3) means that a diffeomorphism along n is related to a conformal transformation for the metric at ∂M. Thus, the boundary condition (2.3) is in this sense "holographic"[1]. One of the differences between our approach and the counterterms method of [4, 5] is that here, instead of fixing the metric at the boundary, we fix the extrinsic curvature. This allows us to readily generalize our results for all odd dimensions.

In order to show this construction explicitly, as well as to deal with this problem in higher dimensions, it is useful to employ the first order formalism in terms of the spin connection $\omega^{ab} = \omega^{ab}_\mu dx^\mu$, and the vielbein $e^a = e^a_\mu dx^\mu$. In this language, the action (2.1) reads

$$I_5 = \kappa\int\limits_M \epsilon_{abcde}R^{ab}R^{cd}e^e + \frac{2}{3l^2}R^{ab}e^ce^de^e + \frac{1}{5l^4}e^ae^be^ce^de^e + dB_4, \tag{2.6}$$

[1] A submanifold that satisfies this condition is also known as *totally umbilical* [20]

where the wedge products between forms is understood, and $R^{ab} \equiv d\omega^{ab} + \omega^a_{\ c}\omega^{cb}$ is the curvature two-form. Henceforth the AdS radius is chosen to be $l = 1$. The Lagrangian can be seen as the Chern-Simons form for the AdS group in five dimensions [10].

The variation of (2.6) yields the field equations plus a surface term Θ

$$\delta I_5 = \kappa \int_M \mathcal{E}_a \delta e^a + 2\mathcal{E}_{ab}\delta\omega^{ab} + \int_{\partial M} \Theta \tag{2.7}$$

with

$$\mathcal{E}_a = \epsilon_{abcde}\bar{R}^{ab}\bar{R}^{de} \ , \tag{2.8}$$

$$\mathcal{E}_{ab} = \epsilon_{abcde}\bar{R}^{cd}T^e \ , \tag{2.9}$$

where $\bar{R}^{ab} := R^{ab} + e^a e^b$, and $T^a = de^a + \omega^a_{\ b}e^b$ is the torsion. Clearly, Θ contains two parts

$$\Theta = \alpha_4 + \delta B_4 \tag{2.10}$$

where α_4 is the boundary term coming from the variation of the bulk action,

$$\alpha_4 = 2\kappa\epsilon_{abcde}\delta\omega^{ab}e^c\left(R^{de} + \frac{1}{3}e^d e^e\right) . \tag{2.11}$$

Thus, Eq. (2.10) can be expressed as

$$\Theta = 2\kappa \int_{\partial M} \epsilon_{abcdf}\delta\theta^{ab}e^c\left(\widetilde{R}^{df} + \left(\theta^2\right)^{df} + \frac{1}{3}e^d e^f\right) + \delta B_4, \tag{2.12}$$

where $\left(\theta^2\right)^{ab} = \theta^a_{\ c}\theta^{cb}$ (see Appendix A.1). Here θ^{ab} is the second fundamental form which plays the role of the extrinsic curvature, and $\widetilde{R}^{ab}$ is the intrinsic curvature of the boundary, which doesn't have normal components (*i.e.* $\widetilde{R}^{ab}n_b = 0$), and is related to the curvature R^{ab} through the Gauss-Codazzi equations (see Appendix A.1).

2.1 Boundary Conditions

The action attains an extremum only if Θ vanishes, which determines the variation of the boundary term δB_4. We look for a boundary condition that allows to obtain B_4 as a local function of the fields at the boundary without using background fields. A boundary condition that satisfies these requirements is

$$\delta\theta^{[ab}e^{c]} = \theta^{[ab}\delta e^{c]} \ , \tag{2.13}$$

and it can be checked that the boundary conditions (2.2) with (2.3) are included in this set. In other words, the boundary conditions (2.2) with (2.3), are a sufficient condition for the

boundary condition (2.13) to be satisfied. By appropriately splitting the integrand in (2.12), and using Eq. (2.13), δB_4 can be written as

$$\begin{aligned}\delta B_4 = -\int\limits_{\partial M} \kappa\epsilon_{abcde} \Big[&\delta\theta^{ab} e^c \left(\widetilde{R}^{de} + \frac{3}{2}\left(\theta^2\right)^{de} + \frac{1}{6} e^d e^e \right) \\ &+ \theta^{ab}\delta e^c \left(\widetilde{R}^{de} + \frac{1}{2}\left(\theta^2\right)^{de} + \frac{1}{2} e^d e^e \right) \Big] , \end{aligned} \tag{2.14}$$

which can be readily integrated as [21]

$$B_4 = -\kappa\epsilon_{abcde}\theta^{ab} e^c \left(\widetilde{R}^{de} + \frac{1}{2}\left(\theta^2\right)^{de} + \frac{1}{6} e^d e^e \right) , \tag{2.15}$$

and in components reduces to (2.4).

To summarize, the action principle in five dimensions is given by (2.6), with the boundary term (2.15). The variation of the action evaluated on a solution of the field equations is

$$\delta I_5 = \kappa \int\limits_{\partial M} \epsilon_{abcde} \left(\delta\theta^{ab} e^c - \theta^{ab}\delta e^c \right) \left(\tilde{R}^{de} + \frac{1}{2}\left(\theta^2\right)^{de} + \frac{1}{2} e^d e^e \right) , \tag{2.16}$$

which explicitly shows that it attains an extremum for the boundary conditions (2.13).

As it is shown in the next section, this boundary condition ensures the existence of a boundary term which is a local function of the fields at the boundary, without using background fields, so that the higher-dimensional Chern-Simons AdS action does have an extremum on-shell. The well-defined action principle obtained so leads to the correct results for the black hole thermodynamics, and also allows to express the conserved charges as surface integrals in a straightforward way.

3. Generalization to $d = 2n + 1$ dimensions

A Chern-Simons action for gravity in $2n + 1$ dimensions that generalizes (2.1) is a linear combination of the form

$$I_{2n+1} = \kappa \int\limits_M \sum_{p=0}^{n} \alpha_p L^{(p)} + \int\limits_{\partial M} B_{2n}, \tag{3.1}$$

where $\kappa = \left(2\,(d-2)!\Omega_{d-2} G_n\right)^{-1}$, and $L^{(p)}$ are the dimensional continuations of Euler densities from lower dimensions,

$$L^{(p)} = \epsilon_{a_1 \ldots a_d} R^{a_1 a_2} \ldots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \ldots e^{a_d} ,$$

and

$$\alpha_p = \frac{l^{2(p-n)}}{d-2p} \binom{n}{p} \tag{3.2}$$

It is useful to rewrite the series (3.1) in terms of an integral over the continuous parameter t

$$I_{2n+1}=\kappa\int\limits_{M}\int\limits_{0}^{1}dt\;\epsilon_{a_1\ldots a_{2n+1}}R_t^{a_1a_2}\ldots R_t^{a_{2n-1}a_{2n}}e^{a_{2n+1}}+\int\limits_{\partial M}B_{2n} \tag{3.3}$$

where $R_t^{ab}:=R^{ab}+t^2e^ae^b$. Following the same procedure as in five dimensions the variation of the boundary term B_{2n} in all odd dimensions is found to be

$$\delta B_{2n}=-\kappa n\int\limits_{0}^{1}dt\;\epsilon\delta\theta e\left(\widetilde{R}+\theta^2+t^2e^2\right)^{n-1}, \tag{3.4}$$

where we have simplified the notation omitting the indices, which are hereafter assumed to be contracted in the canonical way. Using the same set of boundary conditions defined in Eq. (2.13), the boundary term for Chern-Simons AdS gravity in all odd dimensions is found to be

$$B_{2n}=-\kappa n\int\limits_{0}^{1}dt\int\limits_{0}^{t}ds\;\epsilon\theta e\left(\widetilde{R}+t^2\theta^2+s^2e^2\right)^{n-1}. \tag{3.5}$$

With this formula for the boundary term, the finite action (3.3) now reads,

$$I_{2n+1}=\kappa\int\limits_{M}\int\limits_{0}^{1}dt\;\epsilon R_t^n e^{a_{2n+1}}+\int\limits_{\partial M}B_{2n}\;. \tag{3.6}$$

It is straightforward to check that the variation of I_{2n+1}, evaluated on a solution of the field equations,

$$\delta I_{2n+1}=\kappa n\int\limits_{\partial M}\int\limits_{0}^{1}dt\,t\;\epsilon\left(\delta\theta e-\theta\delta e\right)\left(\widetilde{R}+t^2\theta^2+t^2e^2\right)^{n-1},$$

vanishes for the boundary conditions (2.13).

It turns out that the action principle proposed here is finite for configurations that satisfy the boundary conditions and the Euclidean continuation of the action correctly describes the thermodynamics of the system in the canonical ensemble, as it can be seen in the next section.

4. Applications to Black Hole Thermodynamics

Chern-Simons-AdS gravity possesses static black hole solutions, that are asymptotically locally AdS, and whose line element is [22, 23, 24]

$$ds^2=-\Delta^2(r)dt^2+\frac{dr^2}{\Delta^2(r)}+r^2d\Sigma_{d-2}^2 \tag{4.1}$$

with

$$\Delta^2(r)=\gamma-\sigma+r^2. \tag{4.2}$$

Here $d\Sigma_{d-2}^2$ is the line element of the $(d-2)$-dimensional base manifold[2] whose curvature is related to $\gamma = 1, 0, -1$, and the horizon is located at $r_+ = \sqrt{\sigma - \gamma}$. Requiring smoothness of the Euclidean black hole solution at the horizon fixes the period of the Euclidean time as

$$\beta = T^{-1} = \left(\frac{1}{4\pi} \left. \frac{d\Delta^2}{dr} \right|_{r_+} \right)^{-1} = \frac{2\pi}{r_+} \tag{4.3}$$

where T is the black hole temperature.

In the semiclassical approximation, the partition function is given by $Z \approx e^{I_{2n+1}^E}$, where I_{2n+1}^E is the Wick-rotated version of the action (3.6). For fixed temperature, the Euclidean action is related to the free energy in the canonical ensemble, $I_E = -\beta F = S - \beta M$, which defines the mass and entropy of the black hole. For the spherically symmetric case ($\gamma = 1$), the boundary term (3.5) evaluated for the solution (4.1) gives a finite contribution plus a divergent piece,

$$\begin{aligned}\int\limits_{\partial M} B_{2n}^E =& \frac{\beta}{G_n} n (1-\sigma) \int\limits_0^1 dt\, t \left(1 - t^2 (1-\sigma)\right)^{n-1} \\ &+ 2(d-2)!\Omega_{d-2} \beta \kappa n \left[r^2 \int\limits_0^1 dt \left(\sigma + \left(t^2 - 1\right) r^2 \right)^{n-1} \right]^{r=\infty} .\end{aligned} \tag{4.4}$$

Analogously, the bulk term takes the form

$$I_{2n+1}^E = (d-2)!\Omega_{d-2} \beta \kappa \left[2nr^2 \int\limits_0^1 dt \left(t^2 - 1\right) \left(\sigma + \left(t^2 - 1\right) r^2 \right)^{n-1} \right. \tag{4.5}$$

$$\left. + \int\limits_0^1 dt \left(\sigma + \left(t^2 - 1\right) r^2 \right)^n \right]_{r=r_+}^{r=\infty} , \tag{4.6}$$

which also has a divergent piece at infinity, plus a finite term coming from the horizon. It it easy to check that the divergent pieces from both expressions cancel out, leaving a finite Euclidean action,

$$I_{2n+1}^E = \frac{\beta}{G_n} n r_+^2 \int\limits_0^1 dt \left(\sigma + \left(t^2 - 1\right) r_+^2 \right)^{n-1} + \frac{\beta}{G_n} n (1-\sigma) \int\limits_0^1 dtt \left(1 - t^2 (1-\sigma)\right)^{n-1} , \tag{4.7}$$

where the first term comes from the horizon and the second from infinity. The mass is then found to be

[2] Requiring the existence of asymptotic Killing spinors restricts the base manifold to be an Einstein space of Euclidean signature satisfying $R_{mn} = \gamma(d-3)g_{mn}$ admitting at least one Killing spinor [25]. These manifolds are classified, and the constant γ can be normalized to $\pm 1, 0$ by a suitable coordinate rescaling.

$$M=-\frac{\partial I_{2n+1}}{\partial\beta}=\frac{\sigma^{n}-1}{2G_{n}}, \tag{4.8}$$

while the entropy reads

$$S=\left(1-\beta\frac{\partial}{\partial\beta}\right)I_{2n+1} \tag{4.9}$$

$$=\frac{2\pi}{G_{n}}nr_{+}\int_{0}^{1}dt\left(1+t^{2}r_{+}^{2}\right)^{n-1}, \tag{4.10}$$

which, by means of an appropriate change of variable, leads to the expression

$$S=\frac{2\pi}{G_{n}}n\int_{0}^{r_{+}}dr\left(1+r^{2}\right)^{n-1}. \tag{4.11}$$

Note that, as expected, the term coming from infinity in Eq. (4.7) is identified as $-\beta M$, while the term from the horizon is the entropy, as found by different methods [26, 22, 27, 28, 29].

Analogously, for the topological black holes (4.1), the mass is

$$M=\frac{\Sigma_{\gamma}}{2\Omega_{d-2}G_{n}}\left[\sigma^{n}-\gamma^{n}\right], \tag{4.12}$$

where Σ_{γ} is the volume of the base manifold, and the entropy is given by

$$S=\frac{2\pi}{G_{n}}n\frac{\Sigma_{\gamma}}{\Omega_{d-2}}\int_{0}^{r_{+}}dr\left(\gamma+r^{2}\right)^{n-1}, \tag{4.13}$$

which is also in agreement with the Hamiltonian formalism [24]. Note that the mass of the negative constant curvature configurations (locally AdS) depends on the topology of the boundary, and is given by

$$M_{0}=-\frac{\Sigma_{\gamma}}{2\Omega_{d-2}G_{n}}\gamma^{n}, \tag{4.14}$$

which could be interpreted as the Casimir energy of the dual CFT, reflecting the existence of the Weyl anomaly.

5. Conserved Charges as Surface Integrals

The action principle presented here allows to write the conserved charges associated with asymptotic symmetry as surface integrals at infinity in a very straightforward manner. By direct application of Noether's theorem, the conserved current associated with the invariance under diffeomorphisms of the Lagrangian L_{2n+1} is given by (see Appendix A.2)

$$*J=-\Theta-I_{\xi}L_{2n+1} \tag{5.1}$$

where Θ is the boundary term that comes from the variation of the action on shell evaluated for a change in the fields induced by a diffeomorphism, and I_ξ is the contraction operator[3]. The field equations allows to write the current as an exact form, $*J = dQ(\xi)$, and assuming suitable asymptotic conditions for the fields, the conserved charge can be expressed as the surface integral

$$Q(\xi) = \int\limits_{\partial\Sigma} \left(I_\xi \theta^{ab} \frac{\delta L_{2n+1}}{\delta R^{ab}} + I_\xi \theta^{ab} \frac{\delta B_{2n}}{\delta \theta^{ab}} + I_\xi e^a \frac{\delta B_{2n}}{\delta e^a} \right). \tag{5.2}$$

This defines a conserved charge when the parameter ξ is an asymptotic Killing vector. In order to write (5.2) we have demanded that the connection at the boundary be left unchanged by a displacement along ξ, that is, $\mathcal{L}_\xi \omega = 0$, which is not an additional requirement. For the action (3.3), the charge is given by

$$Q(\xi) = \kappa n \int\limits_{\partial\Sigma} \int\limits_0^1 dt t \epsilon \left(I_\xi \theta e + \theta I_\xi e \right) \left(\widetilde{R} + t^2 \theta^2 + t^2 e^2 \right)^{n-1}. \tag{5.3}$$

As the black hole solutions (4.1) possess a timelike Killing vector ∂_t, the mass can also be evaluated as $Q(\partial_t) = M$. For instance, for the spherically symmetric solution ($\gamma = 1$), Eq. (5.3) yields (see Appendix A.3)

$$\begin{aligned} Q(\partial_t) &= 2\kappa n \int\limits_{\partial\Sigma} \int_0^1 dt\ t \epsilon_{01m_1 \ldots m_{d-2}} (\theta_t^{01} e^{m_1} + e_t^0 \theta^{1m_1}) [1 - t^2(1-\sigma)]^{n-1} \tilde{e}^{m_2} \ldots \tilde{e}^{m_{d-2}} \\ &= \frac{n(1-\sigma)}{G_n} \int\limits_0^1 dt t \left(1 - t^2(1-\sigma)\right)^{n-1} = \frac{1}{2G_n} (\sigma^n - 1), \end{aligned} \tag{5.4}$$

which agrees with the previous result (4.8). In general, for all values of γ, one finds

$$Q(\partial_t) = \frac{\Sigma_\gamma}{2\Omega_{d-2} G_n} \left[\sigma^n - \gamma^n \right], \tag{5.5}$$

in agreement with (4.12). We should note that the integrand in the first expression for $Q(\partial_t)$ in (5.4) does not depend on r. Therefore, the mass could be obtained integrating on a surface $\partial\Sigma$ of any radius.

It is worth mentioning that although the mass has been computed following two radically different approaches, the zero point energy (the mass of the locally AdS solutions, $\sigma = 0$) or Casimir energy is the same in both cases.

[3]The action of the contraction operator I_ξ over a p-form $\alpha_p = \frac{1}{p!} \alpha_{\mu_1 \ldots \mu_p} dx^{\mu_1} \ldots dx^{\mu_p}$ is given by $I_\xi \alpha_p = \frac{1}{(p-1)!} \xi^\nu \alpha_{\nu\mu_1 \ldots \mu_{p-1}} dx^{\mu_1} \ldots dx^{\mu_{p-1}}$. In terms of this operator, the Lie derivative reads $\mathcal{L}_\xi = dI_\xi + I_\xi d$.

6. Discussion

Imposing boundary conditions for the curvature instead of the metric in AdS gravity also yields well-defined action principles for different AdS gravity theories in even dimensions [30, 31, 28]. In fact, it can also be shown that the same boundary term (3.5) can render the Einstein-Hilbert action in $2n+1$ dimensions finite,

$$I_{EH\ 2n+1}^{Reg} = \int\limits_{M} \epsilon Re^{2n-1} + \alpha \int\limits_{\partial M} B_{2n} \ , \tag{6.1}$$

for an appropriate α. In the same spirit, the present approach suggests that it is possible to answer the same question in Gauss-Bonnet extended gravity in odd dimensions through a similar set of boundary conditions.

The kind of theories considered here are generalizations of the Einstein-Hilbert action containing terms with higher powers in the curvature. For instance, the five-dimensional theory has a bulk term given by a particular combination of the Einstein-Hilbert action with negative cosmological constant and a Gauss-Bonnet term (2.1). The relevance of these theories has been emphasized in brane-world scenarios (see, e.g., Refs. [32]), and conserved charges have also been obtained in this case following different approaches [33]. Further interesting issues related to Chern-Simons AdS gravity have been explored in Refs. [34]

Here we have constructed a finite action principle for Chern-Simons AdS gravity given by (3.6). This is found requiring the action to attain an extremum for the set of boundary conditions (2.13). This method does not invoke a background configuration and it works for all odd dimensions. The Euclidean continuation of the action was shown to correctly describe the black hole thermodynamics in the canonical ensemble. It is worth mentioning that in our case, the zero point energy (the mass of the locally AdS configurations M_0) becomes fixed and depends on topology of the boundary as in (4.14), in analogy with what occurs for the Einstein-Hilbert action with the counterterms method [5, 35]. This could be interpreted as the Casimir energy of the dual CFT, signaling the presence of the Weyl anomaly. These anomalies have been recently computed for Chern-Simons AdS gravity in Ref. [36]. The action principle presented here also allows readily to construct background-independent conserved charges as surface integrals associated with the asymptotic symmetries, by direct application of Noether's theorem. Alternative conserved charges have been obtained for Chern-Simons AdS gravity by different methods in Refs. [37, 38, 36]. It would be interesting to examine the relationship between these expressions, as well as to explore what would be obtained through time-honored hamiltonian methods [39], as well as with different recent approaches [5, 40].

It is also worthwhile to observe that the action studied in this paper can be regarded as a transgression form, which seems to be the ultimate reason behind its good properties [41].

A. Appendices

A.1 The second fundamental form and the Gauss-Codazzi equations

When one deals with local orthonormal frames $e^a = e^a_\mu dx^\mu$ on a bounded manifold M, the role of the extrinsic curvature K_{ij} is played by the second fundamental form θ^{ab}, which can be defined as follows. Let us consider a manifold $\bar{M}$ which is cobordant with M, i.e., $\partial M = \partial \bar{M}$. If $\bar{M}$ is endowed with a metric that matches the metric of M at the boundary, then the second fundamental form θ^{ab} is defined as the difference between the spin connections of M and $\bar{M}$ at the boundary (see e.g. [42]),

$$\theta^{ab} = \Big[\omega^{ab} - \bar{\omega}^{ab}\Big]_{\partial M} \ . \tag{A.1}$$

It is important to stress that $\bar{\omega}^{ab}$ is the spin connection of the cobordant manifold $\bar{M}$, and has nothing to do with the usual concept of background field. Indeed, in contrast with a background field, the connection $\bar{\omega}^{ab}$ has no meaning in the bulk of M. Hence, as $\bar{\omega}^{ab}$ is defined only at the boundary, it can be naturally identified as the connection for an auxiliary product manifold $\bar{M}$ that is cobordant to M.

The relationship between the second fundamental form and the extrinsic curvature becomes clear using Gaussian coordinates, which can always be defined in an open neighborhood near the boundary. In the vicinity of the boundary, the metric of the spacetime M can be written in Gaussian coordinates as

$$ds^2 = dz^2 + h_{ij}(z,x)dx^i dx^j \ , \tag{A.2}$$

where z is the coordinate normal to the boundary, defined by the surface $z = 0$. Analogously, the coordinates of the cobordant manifold $\bar{M}$ can be chosen so that the metric reads

$$d\bar{s}^2 = dz^2 + h_{ij}(z=0,x)dx^i dx^j \ , \tag{A.3}$$

which matches the metric (A.2) at the boundary. Using the decomposition $a = \{1, \underline{i}\}$ for the tangent space indices and $\mu = \{z, j\}$ for the world indices, the vielbein near the boundary of M can be chosen as

$$e^1 = dz \ ; \ e^{\underline{k}} = e^{\underline{k}}_j(z,x)dx^j \ , \tag{A.4}$$

where $h_{ij} = \eta_{\underline{k}\,\underline{l}} e^{\underline{k}}_i e^{\underline{l}}_j$. Analogously for $\bar{M}$ we choose $\bar{e}^1 = e^1$, and $\bar{e}^{\underline{i}} = e^{\underline{i}}(z=0,x)$. As the second fundamental form is defined at the boundary, it has no components along dz, i.e., $\theta^{ab} = \theta^{ab}_j dx^j$. The corresponding components of the spin connections ω^{ab} and $\bar{\omega}^{ab}$ are obtained from the vanishing of the torsion. Since $\bar{M}$ is a product manifold, $\bar{\omega}^{ab}$ does not have normal components, and hence the only non vanishing components are given by $\bar{\omega}^{\underline{ij}} = \omega^{\underline{ij}}_k(z=0,x)dx^k$, which means that $\theta^{\underline{ij}} = \left(\omega^{\underline{ij}} - \bar{\omega}^{\underline{ij}}\right)\big|_{\partial M} = 0$, and $\theta^{1\underline{i}} = \omega^{1\underline{i}}_j dx^j$, with $\omega^{1\underline{i}}_j = -\frac{1}{2}h_{jk,z}e^{\underline{i}k}$. On the other hand, since the extrinsic curvature is defined as $K_{jk} = \nabla_j n_k$, where n_k is normal to the boundary whose only nonvanishing component is $n_z = 1$, one obtains that

$$K_{jk} = \partial_j n_k - \Gamma^{\mu}_{jk} n_\mu = \frac{1}{2} h_{jk,z} . \tag{A.5}$$

Therefore,

$$\begin{aligned} \theta^{1\underline{i}} &= -K_j^{\ k} e^{\underline{i}}_{k} dx^j \\ \theta^{\underline{ij}} &= 0 \, . \end{aligned} \tag{A.6}$$

expresses the relation between the second fundamental form and the extrinsic curvature.

Note that the curvature two-form R^{ab} can also be decomposed in tangent and normal components defined by

$$\begin{aligned} R^{\underline{ij}} &= d\omega^{\underline{ij}} + \omega^{\underline{i}}_{\ \underline{k}} \omega^{\underline{kj}} + \omega^{\underline{i}}_{\ 1} \omega^{1\underline{j}} \, , \\ R^{1\underline{i}} &= d\omega^{1\underline{i}} + \omega^{1}_{\ \underline{k}} \omega^{\underline{ki}} \, , \end{aligned}$$

which at the boundary can be expressed as

$$\begin{aligned} R^{\underline{ij}} &= d\overline{\omega}^{\underline{ij}} + \overline{\omega}^{\underline{i}}_{\ \underline{k}} \overline{\omega}^{\underline{kj}} + \theta^{\underline{i}}_{\ 1} \theta^{1\underline{j}} \, , \\ R^{1\underline{i}} &= d\theta^{1\underline{i}} + \overline{\omega}^{\underline{i}}_{\ \underline{k}} \theta^{1\underline{k}} \, . \end{aligned}$$

This allows to write the Gauss-Codazzi equations in terms of the second fundamental form

$$R^{\underline{ij}} = \widetilde{R}^{\underline{ij}} + \left(\theta^2\right)^{\underline{ij}} \, , \tag{A.7}$$

$$R^{1\underline{i}} = \tilde{D} \theta^{1\underline{i}} \, , \tag{A.8}$$

where $\tilde{D}$ and $\widetilde{R}^{\underline{ij}}$ are the Lorentz covariant derivative, and the curvature two-form of ∂M, respectively. Hence, the decomposition (A.7) recovers the well-known tensorial form of Gauss-Codazzi relations

$$R^{ij}_{kl} = \widetilde{R}^{ij}_{kl} - K^i_k K^j_l + K^i_l K^j_k \tag{A.9}$$

where R^{ij}_{kl} and $\widetilde{R}^{ij}_{kl}$ the Riemann tensors of the bulk and the boundary metrics, respectively.

As an application, note that since the boundary ∂M located at fixed z, the components of R^{ab} and e^a along dz do not contribute to the expression (2.11). Hence, using the Gauss-Codazzi equations (A.7), the boundary term (2.10) reads

$$\Theta = 4\kappa \int\limits_{\partial M} \epsilon_{1\underline{ijkl}} \delta\theta^{1\underline{i}} e^{\underline{j}} \left(\widetilde{R}^{\underline{kl}} + \left(\theta^2\right)^{\underline{kl}} + \frac{1}{3} e^{\underline{k}} e^{\underline{l}} \right) + \delta B_4 \, , \tag{A.10}$$

which by virtue of Eq. (A.6) it can be covariantized back as

$$\Theta = 2\kappa \int\limits_{\partial M} \epsilon_{abcdf} \delta\theta^{ab} e^{c} \left(\widetilde{R}^{df} + \left(\theta^2\right)^{df} + \frac{1}{3} e^{d} e^{f} \right) + \delta B_4 \, . \tag{A.11}$$

Therefore, the net effect is that the connection $\bar{\omega}^{ab}$ can be regarded as being just a reference field fixed at the boundary, so that $\delta\omega^{ab}$ can be replaced by $\delta\theta^{ab}$. Consistently, the curvature at the boundary $\widetilde{R}^{\underline{ij}} = \widetilde{R}^{\underline{ij}}(\bar{\omega})$ possesses a vanishing variation, so that the variation for the curvature two-form $R^{\underline{ij}}$ is given by

$$\delta R^{\underline{ij}} = D\delta\omega^{\underline{ij}} = \delta\left(\theta^2\right)^{\underline{ij}} \ . \tag{A.12}$$

Thus, in higher odd dimensions one proceeds in the same way. The surface term Θ now reads

$$\Theta = \alpha_{2n} + \delta B_{2n} \tag{A.13}$$

where α_{2n} is the boundary term coming from the variation of the bulk term in Eq. (3.1)[4],

$$\alpha_{2n} = \kappa n \int_0^1 dt \epsilon_{a_1 \ldots a_{2n+1}} \delta\omega^{a_1 a_2} e^{a_3} R_t^{a_4 a_5} \ldots R_t^{a_{2n} a_{2n+1}}, \tag{A.14}$$

which can be expressed as

$$\alpha_{2n} = \kappa n \int_0^1 dt \epsilon \delta\theta e \sum_{k=0}^{n-1} C_k^{n-1} \left(\widetilde{R} + t^2 e^2\right)^{n-1-k} \theta^{2k} \ , \tag{A.15}$$

with $C_p^n = \begin{pmatrix} n \\ p \end{pmatrix}$. It is simple to see that the boundary condition (2.13) makes it possible to integrate B_{2n} from its variation as in the five-dimensional case, since Eq. (2.13) allows to express α_{2n} as

$$\begin{aligned} \alpha_{2n} = & \ \kappa n \int_0^1 dt \epsilon \delta\theta e \sum_{k=0}^{n-1} C_k^{n-1} \left(\widetilde{R} + t^2 e^2\right)^{n-1-k} \theta^{2k} \left(1 - t^{2k+1}\right) \\ & + \kappa n \int_0^1 dt \epsilon \theta \delta e \sum_{k=0}^{n-1} C_k^{n-1} \left(\widetilde{R} + t^2 e^2\right)^{n-1-k} t^{2k+1} \theta^{2k}. \end{aligned} \tag{A.16}$$

After some algebraic manipulation, that includes identities similar to [21] in five dimensions, we integrate out the variation of θ and e as

$$\begin{aligned} \delta I_{2n+1} = & \int_{\partial M} \kappa n \int_0^1 dt \epsilon \sum_{k=0}^{n-1} C_k^{n-1} \sum_{l=0}^{n-1-k} C_l^{n-1-k} \widetilde{R}^{n-1-k-l} \int_0^t ds t^{2k+1} \delta\theta^{2k+1} s^{2l} e^{2l+1} \\ & + \kappa n \int_0^1 dt \epsilon \sum_{k=0}^{n-1} C_k^{n-1} \sum_{l=0}^{n-1-k} C_l^{n-1-k} \widetilde{R}^{n-1-k-l} \int_0^t ds t^{2l+1} \theta^{2l+1} s^{2k} \delta e^{2k+1} \\ & + \delta B_{2n}. \end{aligned} \tag{A.17}$$

[4]Note that in the last expression it is easy to recognize the Lagrangian of CS-AdS gravity for the odd dimension right below: $\alpha_{2n} = \kappa n \epsilon_{a_1 \ldots a_{2n+1}} \delta\omega^{a_1 a_2} L_{2n-1}^{a_3 \ldots a_{2n+1}}(R, e)$.

Finally, the boundary term B_{2n} can be integrated in a very concise form as the double integral in Eq. (3.5).

A.2 Noether Theorem

In order to fix the notation and conventions, here we briefly review Noether's theorem. Consider a d-form Lagrangian $L(\varphi, d\varphi)$, where φ denotes collectively a set of p-form fields. An arbitrary variation of the action under a local change $\delta\varphi$ is given by the integral of

$$\delta L = (E.O.M)\delta\varphi + d\Theta(\varphi, \delta\varphi), \tag{A.18}$$

where E.O.M. stands for equations of motion and Θ is the corresponding boundary term [43]. The total change in φ ($\bar{\delta}\varphi = \varphi'(x') - \varphi(x)$) can be decomposed as a sum of a local variation and the change induced by a diffeomorphism, that is, $\bar{\delta}\varphi = \delta\varphi + \mathcal{L}_\xi\varphi$, where $\mathcal{L}_\xi$ is the Lie derivative operator. In particular, a symmetry transformation acts on the coordinates of the manifold as $\delta x^\mu = \xi^\mu(x)$, and on the fields as $\delta\varphi$, leading to a change in the Lagrangian given by $\delta L = d\Omega$.

Noether's theorem states that there exists a conserved current given by

$$*J = \Omega - \Theta(\varphi, \delta\varphi) - I_\xi L, \tag{A.19}$$

which satisfies $d * J = 0$. This, in turn, implies the existence of the conserved charge

$$Q = \int\limits_\Sigma *J, \tag{A.20}$$

where we assume a manifold with topology $M = R \times \Sigma$ and Σ is the spatial section of the manifold. If the Lagrangian is supplemented by a boundary term $\alpha(\varphi, \overline{\varphi})$ that contains a dependence on a fixed field $\overline{\varphi}$ at ∂M, the current derived from the Noether theorem takes the form

$$*J' = dQ + \frac{\delta\alpha}{\delta\overline{\varphi}}\mathcal{L}_\xi\overline{\varphi}. \tag{A.21}$$

A sufficient condition to ensure the conservation of the current is taking $\mathcal{L}_\xi\overline{\varphi} = 0$.

A.3 Second fundamental form and the curvature of a black hole

For the black holes considered here, the line element is given by

$$ds^2 = \Delta(r)^2 dt^2 + \frac{dr^2}{\Delta(r)^2} + r^2 d\Sigma_\gamma^2 \tag{A.22}$$

where $d\Sigma_\gamma^2$ is the line element of the $(d-2)$-dimensional base manifold Σ_γ with constant curvature $\gamma = \pm 1, 0$. The corresponding vielbeins can be chosen as

$$\begin{aligned} e^0 &= \Delta(r)dt \\ e^1 &= \frac{dr}{\Delta(r)} \\ e^n &= r\tilde{e}^n \end{aligned}$$

where $\tilde{e}^n$ corresponds to the vielbein of the base manifold Σ_γ. If the cobordant manifold is endowed with a product metric that matches (A.22) at the boundary, we have

$$\begin{aligned}
\theta^{01} &= \frac{1}{2}(\Delta^2)' dt \\
\theta^{1m} &= -\frac{\Delta(r)}{r} e^m = -\Delta(r)\tilde{e}^m
\end{aligned}$$

where the prime means radial derivative ∂_r. The curvature two-form is given by

$$\begin{aligned}
R^{01} &= -\frac{1}{2}(\Delta^2)'' e^0 e^1 \ , \\
R^{0m} &= -\frac{(\Delta^2)'}{2r} e^0 e^m \ , \\
R^{1m} &= -\frac{(\Delta^2)'}{2r} e^1 e^m \ , \\
R^{mn} &= (\gamma - \Delta^2)\tilde{e}^m \tilde{e}^n \ .
\end{aligned}$$

Acknowledgments

We wish to thank M. Bañados, G. Barnich, M. Hassaïne, M. Henneaux, G. Kofinas, K. Skenderis, and S. Theisen for helpful discussions. PM is grateful to the Centro de Estudios Científicos (CECS) for the hospitality during the development of this work, and to the International Center of Theoretical Physics, ICTP-Trieste, which made his visits to Valdivia possible. RO thanks Prof. M. Henneaux for kind hospitality at the Université Libre de Bruxelles. This work was partially funded by FONDECYT grants 1010449, 1010450, 1020629, 1040921, 3030029, and 7010450 from FONDECYT. Institutional support to CECS from Empresas CMPC is also acknowledged. CECS is a Millennium Science Institute and is funded in part by grants from Fundación Andes and the Tinker Foundation.

References

[1] G. Gibbons and S.W. Hawking, Phys. Rev. **D15** (1977) 2752.

[2] O. Aharony, S. Gubser, J.Maldacena, H. Ooguri, and Y. Oz, Phys. Rept. **323** (2000) 183, e-Print: hep-th/9905111.

[3] S.W. Hawking and D. Page Commun. Math. Phys. **87 (**1983) 577.

[4] M. Henningson and K. Skenderis, JHEP **9807** (1998) 023, e-print: hep-th/9806087.

[5] V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208** (1999) 413, e-print: hep-th/9902121.

[6] A.M. Awad and C.V. Johnson, Phys. Rev. **D63** (2001) 124023, e-print: hep-th/0008211.

[7] R. Clarkson, L. Fatibene and R.B. Mann. Nucl. Phys. **B652** (2003) 348, e-print: hep-th/0210280.

[8] D. Lovelock, J. Math. Phys. **12** (1971) 498.

[9] B. Zumino, Phys. Rept. **137** (1986) 109.

[10] A. H. Chamseddine, Phys. Lett. **B233** (1989) 291.

[11] A. Achúcarro and P.K. Townsend, Phys. Lett. **B180**(1986) 89.

[12] P.S. Howe, J.M. Izquierdo, G. Papadopoulos, P.K. Townsend Nucl. Phys. **B467** (1996) 183, e-Printe: hep-th/9505032.

[13] A. H. Chamseddine, Nucl. Phys. **B346** (1990) 213.

[14] M. Bañados, R. Troncoso and J. Zanelli, Phys. Rev. **D54** 2605 (1996), e-Print: gr-qc/9601003.

[15] R. Troncoso and J. Zanelli, Phys. Rev. **D58** (1998) 101703, e-Print: hep-th/9710180.

[16] R. Troncoso and J. Zanelli, Int. Jour. Theor. Phys. **38** (1999) 1181, e-Print: hep-th/9807029. R. Troncoso and J. Zanelli, *Chern-Simons Supergravities with Off-Shell Local Superalgebras*, in "Black Holes and the Structure of the Universe", Santiago, Chile, Aug 1997. C.Teitelboim and J. Zanelli (Eds.) World Scientific, Singapore, 1999, e-Print: hep-th/9902003.

[17] R. Troncoso and J. Zanelli, Class. Quan. Grav. **17** (2000) 4451, e-Print: hep-th/9907109.

[18] M. Hassaïne, R. Olea and R. Troncoso, Phys. Lett. **B599** (2004) 111. e-Print: hep-th/0210116.

[19] M. Hassaïne, R. Troncoso and J. Zanelli, Phys. Lett. **B596** (2004) 132. e-Print: hep-th/0306258.

[20] M. Spivak, *Differential Geometry, A Comprehensive Introduction*, (Publish or Perish, Houston, 1979).

[21] In the expression (2.14), the integration of the term

$$\epsilon_{abcde}\delta\theta^{ab}\theta_f^c\theta^{fd}e^e = \frac{1}{3}\epsilon_{abcde}\delta\left(\theta^{ab}\theta_f^c\theta^{fd}\right)e^e$$

is possible by virtue of the following identity

$$\epsilon_{abcde}\delta\theta^{ab}\theta_f^c\theta^{fd}e^e = \epsilon_{abcde}\theta^{ab}\delta\theta_f^c\theta^{fd}e^e + \frac{1}{2}\epsilon_{abcde}\theta^{ab}\delta\theta^{cd}\theta_f^e e^f \ ,$$

which comes from the invariance of the Levi-Civita tensor under Lorentz transformations. Since the non-vanishing components of the second fundamental form require the presence of a normal direction in tangent space (see Eq. (A.6)), the last term identically vanishes.

[22] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. **D49** (1994) 975, e-Print: gr-qc/9307033.

[23] R.-G. Cai and K.-S. Soh, Phys. Rev. **D59** (1999) 0044013, e-Print: gr-qc/9808067.

[24] R. Aros, R. Troncoso and J. Zanelli, Phys. Rev. **D63** (2001) 084015, e-Print: hep-th/0011097.

[25] R. Aros, C. Martínez, R. Troncoso and J. Zanelli, JHEP **0205** (2002) 020, e-Print: hep-th/0204029

[26] R. C. Myers and J. Z. Simon, Phys. Rev. **D38** (1988) 2434 .

[27] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **72** (1994) 957, e-Print: gr-qc/9309026.

[28] J. Crisóstomo, R. Troncoso and J. Zanelli, Phys. Rev. **D62**, (2000) 084013, e-Print: hep-th/0003271.

[29] T. Clunan, S.F. Ross and D.J. Smith, Class. Quant. Grav. **21** (2004) 3447. e-Print: gr-qc/0402044.

[30] R. Aros, M. Contreras, R. Olea, R. Troncoso and J. Zanelli, Phys. Rev. Lett. **84**, (2000) 1647, e-Print: gr-qc/9909015.

[31] R. Aros, M. Contreras, R. Olea, R. Troncoso and J. Zanelli, Phys. Rev. **D62**, (2000) 044002, e-Print: hep-th/9912045.

[32] S. Nojiri, and S. D. Odintsov, JHEP **0007** (2000) 049, e-Print: hep-th/0006232. A. Iglesias and Z. Kakushadze, Int. J. Mod. Phys. **A16** (2001) 3603, e-Print: hep-th/0011111. A. Iglesias and Z. Kakushadze, Phys. Lett. **B515** (2001) 477, e-Print: hep-th/0012049. M. Cvetic, S. Nojiri, and S. D. Odintsov, Nucl. Phys. **B628** (2002) 295, e-Print: hep-th/0112045. Y.M. Cho and I.P. Neupane, Phys. Rev. **D66** (2002) 024044, e-Print: hep-th/0202140. J. E. Lidsey, S. Nojiri and S. D. Odintsov, JHEP **0206** (2002) 026, e-Print: hep-th/0202198. J. Gregory and A. Padilla, Class. Quant. Grav. **20 (**2003) 4221, e-Print: hep-th/0304250. N. Deruelle and M. Sasaki, Prog. Theor. Phys. **110** (2003) 441, e-Print: gr-qc/0306032. N. Deruelle and C. Germani, Nuovo Cim. **118B** (2003) 977. e-print: gr-qc/0306116; G. Kofinas, R. Maartens and E. Papantonopoulos, JHEP **0310** (2003) 066, e-Print: hep-th/0307138. C. Charmousis, S. C. Davis and J. F. Dufaux, JHEP **0312** 029,2003, e-Print: hep-th/0309083. M. Minamitsuji and M. Sasaki, *Linearized gravity on the de Sitter brane in the Einstein Gauss-Bonnet theory*, e-Print: hep-th/0404166.

[33] S. Deser and B. Tekin, Phys. Rev. Lett. 89 (2002) 101101, e-Print: hep-th/0205318. A. Padilla, Class. Quant. Grav. **20 (**2003) 3129, e-Print: gr-qc/0303082. S. Deser and B. Tekin, Phys. Rev. **D67** (2003) 084009, e-Print: hep-th/0212292. N. Deruelle, J. Katz and S. Ogushi, Class. Quant. Grav. **21** (2004) 1971, e-Print: gr-qc/0310098. A. N. Petrov, Vestnik Mosk. Univ. Ser. 3. Fiz. Astron., No. 2, 10 (2004) , e-Print: gr-qc/0401085.

[34] P. Horava, Phys. Rev. **D59** (1999) 046004, e-Print: hep-th/9712130. H. Nastase, *Towards a Chern-Simons M theory of* $OSp(1|32)\times OSp(1|32)$, e-Print: hep-th/0306269. P. Mora, Nucl. Phys. **B594** (2001) 229, e-Print: hep-th/0008180. P. Mora and H. Nishino, Phys. Lett. **B482** (2000) 222, e-Print: hep-th/0002077. O. Chandía, R. Troncoso and J. Zanelli, *Dynamical content of Chern-Simons supergravity*, Published in "Buenos Aires 1998, Trends in theoretical physics 2" 231, e-Print: hep-th/9903204 J. Crisóstomo, S. del Campo and J. Saavedra, *"Hamiltonian treatment of collapsing thin shells in Lanczos-Lovelock's theories"* , e-Print: hep-th/0311259. M. Bañados, Phys. Rev. Lett. **88** (2002) 031301, e-Print: hep-th/0107214. J. Alfaro and M. Bañados, Phys. Rev. **D68** (2003) 085013, e-Print: hep-th/0307249. P. Salgado, F. Izaurieta and E. Rodriguez, Phys. Lett. **B574** (2003) 283, e-Print: hep-th/0305180. P. Salgado, F. Izaurieta and E. Rodríguez, Phys. Lett. B586 (2004) 397, e-Print: hep-th/0402208.; M. Bañados, L. Garay and M. Henneaux, Nucl. Phys. **B476** (1996) 611, e-Print: hep-th/9605159. J. Gegenberg and G. Kunstatter, Phys.Lett. **B478** (2000) 327, e-Print: hep-th/9905228. O. Mišković and J. Zanelli, J. Math. Phys. **44** (2003) 3876, e-Print: hep-th/0302033.

[35] R. Emparan, C.V. Johnson and R.C. Myers, Phys. Rev. **D60**, (1999) 104001, e-print: hep-th/9903238.

[36] M. Bañados, A. Schwimmer and S. Theisen, JHEP **0405** (2004) 039. e-Print: hep-th/0404245.

[37] S. Silva, Nucl. Phys. **B558** (1999) 391, e-Print: hep-th/9809109.

[38] G. Allemandi, M. Francaviglia and M. Raiteri, Class. Quant. Grav. **20** (2003) 5103, e-Print: gr-qc/0308019.

[39] T. Regge and C. Teitelboim, Annals Phys. **88** (1974) 286. M. Henneaux and C. Teitelboim, Commun. Math. Phys. **98** (1985) 391. J. D. Brown and M. Henneaux, Commun. Math. Phys. **104**, 207 (1986). M. Henneaux, *Asymptotically Anti-De Sitter Universes In D = 3, 4 And Higher Dimensions*, Proceedings of the Fourth Marcel Grossmann Meeting on General Relativity, Rome 1985. R. Ruffini (Ed.), Elsevier Science Publishers B.V., pp. 959-966.

[40] A. Ashtekar and S. Das, Class. Quant. Grav. **17** (2000) L17, e-Print: hep-th/9911230. G. Barnich and F. Brandt, Nucl. Phys. **B633** (2002) 3, e-Print: hep-th/hep-th/0111246. J. Gegenberg, C. Martínez and R. Troncoso, Phys. Rev. **D67**, 084007 (2003); G. Barnich, *"Conserved charges in gravitational theories: Contribution from scalar fields"*, in Proceedings of 3rd International Sakharov Conference on Physics, Moscow, Russia, 24-29 Jun 2002. arXiv:gr-qc/0211031; G. Barnich, Class. Quant. Grav. **20**, 3685 (2003), e-Print: hep-th/0301039.

[41] P. Mora, R. Olea, R. Troncoso and J. Zanelli, Preprint CECS-PHY-04/13.

[42] T. Eguchi, P.B. Gilkey and A. J. Hanson, Phys. Rept. **66**, (1980) 213.

[43] P. Ramond, *Field Theory, A Modern Primer*, (Addison-Wesley, Pub. Co., Menlo Park, 1980).